# 2021 Gauss Math Tournament

## Division II Sprint Round

# Instructions

Welcome to the 9$^{th}$ annual Gauss Mathematics Tournament! Please make sure that you are in the correct division. You are about to take the Division III Sprint and Target rounds for students in grades 5-6. If you are not in these grades, please let us know right away and we will help you find your proper division.

You will first take the **Sprint Round**, which will be a 50 minute contest consisting of 40 short-answer problems. The problems are in increasing difficulty order and are worth one point each.

After a short break following the end of the Sprint Round, you will take the **Target Round**, which will consist of 8 problems to be solved in 20 minutes. The problems are in increasing difficulty order and are worth two points each.

The ten highest total scorers on the Sprint and Target rounds will advance to the **Countdown Round**, an exciting head-to-head buzzer contest. More details will be given at the beginning of the Countdown Round.

**You may use a calculator on both the Sprint and Target Rounds.** However, other aids, such as books, notes, other people, magic crystal balls, etc. are prohibited.

**Please read the section below regarding important formatting instructions.** These rules are important to remember while taking the test as you may not receive credit for an improperly formatted answer.

Good luck, and may the odds be ever in your favor!

# Formatting

For both the Sprint and Target Rounds, your answers will be collected on a Google Form. The answer to each question will be a rational number. If your answer is an integer, it should be input as such. For example, if a question asks "What is $1 + 2$?" the correct input is

3

If your answer is a rational number, you should input it as an **improper fraction in lowest terms**. If you answer as a mixed number or decimal, or is not in lowest terms, your answer will be marked wrong. For example, if a question asks "What is 57 divided by 6 in simplest form?" the **only** acceptable answer is:

19/2

The following answers will **not** be accepted:

57/6     9 1/2     9.5

If any answer is negative, simply enter a minus sign (dash) in front of the number, but **do not leave any space between the minus sign and the number**. For example, an answer of $-\frac{3}{4}$ should be input as:

$-3/4$

and not as:

$- \ \ 3/4$

Please keep these rules in mind as you answer the problems!

1. Evaluate $1 + 2 + 3 + \ldots + 15$.
2. Find the area of a square with perimeter 20.
3. The median and unique mode of the set $\{1, 1, 3, 3, 5, x\}$ are the same, where $x$ can be any integer. What is $x$?
4. Stanley can bake 12 cookies every day. How many *hours* will it take Stanley to bake 360 cookies?
5. Lucas is at a farm where there are chickens (which have two legs and one head) and cows (which have four legs and one head). If Lucas counts 15 heads and 48 legs, then how many chickens are at the farm?
6. In a class of 30 students, 17 students like chemistry and 10 students like both chemistry and physics. If there are 4 people in the class who don't like either chemistry or physics, then how many students like physics but don't like chemistry?
7. What is the units digit of $3^{2021}$?
8. Let $x$ be a positive integer, such that $2x$ is eight times the sum of the digits of $x$. Find the smallest possible value of $x$.
9. A right triangle has a hypotenuse of length 20 and a leg of length 12. What is the area of the triangle?
10. Michael is playing with a set of cubes. He has one big cube with side length 2, and 100 small cubes each with side length 1. Michael wants to use the big cube along with some small cubes to build a cube with side length 3. How many small cubes will he need to use?
11. Circle A has an area of 35. Circle B has a radius twice that of circle A. Find the area of Circle B.
12. Andrew is drinking grape juice from a very large cup that is initially three-fourths full. After drinking half of the grape juice from the cup, he then pours in a gallon of grape juice. He again drinks half of the grape juice and then pours in another gallon. Now, the cup is full. How many gallons of grape juice are in the cup?
13. A cook creates pancakes, distributing plastic bags that can contain two or three pancakes. What is the largest number of pancakes that can not be obtained by a combination of the cook's pancakes?
14. What is the sum of the digits of $2^8 \cdot 5^{10}$?
15. Find the sum of all terms less than 1000 that are present in both the arithmetic sequence $4, 9, 14, \ldots$ AND the geometric sequence $3, 6, 12, \ldots$
16. If $f(x) = mx + b$ for some $m < 0$ and $f(x) = 9f^{-1}(x) + 22$, what is $b$?
17. The renowned establishment of Sean's Beans has 3 sizes of servings: 5 beans for \$1, 10 beans for \$1.75, and 30 beans for \$5. How much does it cost to purchase 85 beans in the cheapest way possible?
18. Ben is three years younger than twice of Paden's age; each of these two toddlers' ages is prime. Meanwhile, the square of Michael's age is one more than the product of the ages of Ben and Paden. What is the sum of the boys' three ages?
19. The operation $A \heartsuit B$ satisfies $A \heartsuit B + B \heartsuit A = (AB)^2$ for any positive integers $A$ and $B$. Compute $(3 \heartsuit 1) + (2 \heartsuit 2) + (1 \heartsuit 3)$.
20. A square and a rectangle both have integer side lengths and the same area. If the perimeter of the rectangle is 26, find the perimeter of the square.
21. Three pairs of girl-boy twins gather. The kids are going to be seated close together in one row of chairs. How many ways are there to arrange the children so that each individual can only be next to its twin or a peer of its same gender?

22. Darren is driving at a rate of 30 mph. He realizes halfway through his journey that if he doubles his speed, he will reach his destination in half an hour less than he would if he continued at a speed of 30 mph. How many miles long was his entire journey?

23. Taylor is watching a show where 500 marbles are distributed to seven contestants. What is the minimum number of marbles that the person with the most marbles has?

24. A chef leads his two trainees to taste three new dishes. The chef and his trainees are going to taste two dishes each, such that every dish is tasted by exactly two of the three people. In how many ways can this happen?

25. Rabbit Robert's watch is 1 second fast each hour and Tortoise Tim's watch is 1.5 seconds slow each hour. Right now, both watches show the same time. After how many days will they show the same time again? Assume that the watches can show AM and PM. Round to the nearest day, if necessary.

26. Michael shares the candy that he has collected with his little sister. Three indistinguishable pieces of bubblegum and four indistinguishable chocolate bars are to be divided among the two. How many ways are there for the siblings to split the delicacies among the two of them?

27. The equation $2x^3 + 5x^2 - 111x + 54 = 0$ has three solutions, one of which is $x = 6$ and another one of which is one-twelfth of the first solution. What is the sum of the three solutions?

28. In a paintball match, the blue team has three players and the red team has two players. The players arrange themselves in line from left to right so that each player cannot be right next to any of its teammates. In how many ways can the five players line up?

29. Jennifer can paint a house in 6 days working alone. Jeffrey and Andrew together can paint a house in 4 days. Andrew can paint a house in 4 days if Jennifer helps him for 2 of those days. How many days does it take Jeffrey to paint a house alone?

30. An equilateral triangle is inscribed inside a semicircle of radius three so that one vertex touches the top of the semicircle and one side of the polygon is on the semicircle's diameter. What is the area of the triangle?

31. Jimmy has two lozenges each of three different types (six lozenges total). He decides to take four and give two to Laura. How many ways are there for the two to split the lozenges, where lozenges of the same type are indistinguishable?

32. Adam is currently at the point $(0, 0)$ on the coordinate plane. If Adam is on the point $(x, y)$, he can hop to either $(x + 5, y + 6)$ or $(x + 6, y + 5)$. Brian is at the point $(a, a)$ for some positive integer $a$, such that Adam can reach Brian by hopping. Find the smallest three-digit value of $a$.

33. There is an interesting five-digit number Q. P is the number produced when a 1 is added after Q, and R is the number produced when a 1 is added before P. Knowing that P is three times as large as R, what is Q?

34. Marvelous Mary is a magnificent musician, but she has a quirk - namely, she performs only on days that have at least one 1 as a digit in either the day or the month! Her exasperated manager is now trying to calculate the number of days that Mary will be available to perform. Knowing that Mary performs at most once on each day she can perform, what is the maximum number of times Mary can perform in a non-leap year?

35. The graphs of $g(x) = 2x + 1$ and $f(x) = 0.25 * (g(x))^2 - 3 * g(x) + 9.75$ intersect at two points: $(a, b)$ and $(c, d)$. What is $a + b + c + d$?

36. Given an integer $n$ in base 10, $S(n)$ is the sum of the digits of $n$. Find $S(S(S(S(S(37^{2021})))))$.

37. Alice and Bob took a test, which has ten questions. Given that Alice answered six questions correctly and Bob answered eight questions correctly, compute the probability that there exists at least one question on the test which is answered correctly by Alice but not by Bob.

38. Find the largest integer $n$ for which $5^n$ is a factor of the product of the first 256 positive integers.

39. Triangle $ABC$ has a right angle at $A$, with $AC > AB$. Let $X$ denote the point on segment $BC$ such that $BX = 2CX$. Suppose that the circle with diameter $BX$ is tangent to $AC$. If $BC = 2020$, find the length of $AB$.

40. Allie is at the bottom back left vertex of a unit cube, on the point $(0, 0, 0)$. How many possibilities are there for her to arrive at the top front right vertex, the point $(1, 1, 1)$ of the cube in exactly five moves, such that on each move, she travels to an adjacent vertex?