GAUSS ACADEMY of Mathematical Education

MATH UNITES US

2021 Gauss Math Tournament

Division III Target Round

Instructions

Welcome to the 9th annual Gauss Mathematics Tournament! Please make sure that you are in the correct division. You are about to take the Division III Sprint and Target rounds for students in grades $7-8$. If you are not in these grades, please let us know right away and we will help you find your proper division.

You will first take the **Sprint Round**, which will be a 50 minute contest consisting of 40 short-answer problems. The problems are in increasing difficulty order and are worth one point each.

After a short break following the end of the Sprint Round, you will take the **Target Round**, which will consist of 8 problems to be solved in 20 minutes. The problems are in increasing difficulty order and are worth two points each.

The ten highest total scorers on the Sprint and Target rounds will advance to the **Countdown Round**, an exciting head-to-head buzzer contest. More details will be given at the beginning of the Countdown Round.

You may use a calculator on both the Sprint and Target Rounds. However, other aids, such as books, notes, other people, magic crystal balls, etc. are prohibited.

Please read the section below regarding important formatting instructions. These rules are important to remember while taking the test as you may not receive credit for an improperly formatted answer.

Good luck, and may the odds be ever in your favor!

Formatting

For both the Sprint and Target Rounds, your answers will be collected on a Google Form. The answer to each question will be a rational number. If your answer is an integer, it should be input as such. For example, if a question asks "What is $1+2$?" the correct input is

$$3$$

If your answer is a rational number, you should input it as an **improper fraction in lowest terms**. If you answer as a mixed number or decimal, or is not in lowest terms, your answer will be marked wrong. For example, if a question asks "What is 57 divided by 6 in simplest form?" the **only** acceptable answer is:

$$19/2$$

The following answers will **not** be accepted:

$$57/6 \qquad 9\ 1/2 \qquad 9.5$$

If any answer is negative, simply enter a minus sign (dash) in front of the number, but **do not leave any space between the minus sign and the number**. For example, an answer of $-\frac{3}{4}$ should be input as:

$$-3/4$$

and not as:

$$-\ 3/4$$

Please keep these rules in mind as you answer the problems!

2021 Gauss Math Tournament

Division III (Target Round 20 minutes, 8 Questions)

1. A seven-digit number is ultra-friendly if it satisfies both of the following two conditions:
(1) it contains digits 1,2,3,4,5,6,7 exactly once each;
(2) no three consecutive digits are all odd.
How many ultra-friendly numbers are there?

2. You are playing a game with Cheating Chika. Each of you flips five coins, and you win if you flip strictly more heads than Chika. However, Chika decides to cheat, and one of her coins has both faces heads (her other four coins, and all five of your coins, are fair). What is the probability that you win?

3. A triangle ABC has side lengths AB=7, BC=8, CA=9. Let X be the intersection of the line perpendicular to BC from point A and the median from B to AC. Let D, E, F be the centroids of triangles XBC, XAC, XAB, respectively. Find the square of the area of triangle DEF.

4. Triangle ABC has $\angle A = 60$degrees and $BC = 11$. Points D and E are on sides AB and AC, respectively, such that BD=CE and $DE = 7$. Compute the area of quadrilateral $BDEC$.

5. A triangle with side lengths 3, 4, 5 and a 1x100 rectangle overlap. Find the largest possible area of their intersection.

6. A sequence a_n satisfies $a_1 = 53, a_2 = 7, (53 \cdot 7)({a_i}^2 - a_{i-1}a_{i+1}) = a_i a_{i-1}$for all $i \geq 2$. Compute the first value of $k$so that $a_k = 0$.

7. A random integer n is selected uniformly and at random from 1 to 729, inclusive. Let k be the largest integer for which n/3^k is an integer. Find the expected value of k.

8. Compute the smallest positive integer $k$so that the smallest number with 5^kfactors has a factor of the form a^{10}, where $a$is an integer greater than 1.