## Gauss School and Gauss Math Circle 2019 Gauss Math Tournament

## Grade 5-6 (Sprint Round 50 minutes, 40 Questions)

1. What is the sum of the factors of 12 ?
2. Triangle $A B C$ has a right angle at $A$ and has side lengths $A B=5$, and $A C=12$. What is BC?
3. One bip equals three bops. One bop equals four baps. Two baps equal 5 boops. How many boops are 3 bips equal to?
4. What is the sum of the even numbers between 41 and 51 ?
5. What is the difference between the 5th smallest prime number and the 5th smallest even number?
6. How many permutations are there of the word, "PEMDAS"?
7. How many of the first 10 odd numbers are prime?
8. Andrew has 4 blue socks, 5 white socks, and 17 purple socks. Assuming that he reaches into his drawer without looking, what is the minimum number of socks he must pull out to guarantee he has a pair of same-colored socks?
9. What is the sum of the angles in a pentagon?
10. What is the most likely sum of two dice?
11. Find $a+b+c+d+e$ if $a+b=1, b+c=2, c+d=3, d+e=4$, and $e+a=0$.
12. What is the probability that three rolled dice sum to 17 ?
13. A square of side length 4 is folded in half lengthwise to form a rectangle. What is the length of a diagonal of the resulting shape?
14. On day one, Mary gains one lamb. Each day after that, she gains one more lamb than the previous day. (For example, she gains two lambs on the second day). How many lambs does she have at the end of the ninth day?
15. How many hours would it take for Amelia to fly along the edge of a ring with a radius of $300 / \pi$ kilometers, if she travels at a rate of 5 kilometers every 20 minutes?
16. How many cubes are there between 100 and 1000 (inclusive)?
17. There are positive numbers $a$ and $b$ such that $a+b=10$ and $a b=25$, what is $a^{\wedge} 2+b^{\wedge} 2$ ?
18. If three squadrons of bees can deliver honey to their hive at a rate of two combs an hour, then how many squadrons of bees would it take to deliver 7200 combs in 3 hours?
19. How many prime numbers less than 30 are in the form $4 n+1$, where $n$ is an integer?
20. A regular hexagon is inscribed in a circle of diameter 20, what is the side length of the hexagon?
21. There are 10 people at a party. How many handshakes occur if every pair of people must shake hands exactly once?
22. Given right triangle $A B C$, where $B C$ is the hypotenuse, and square $A B E F$ outside $A B C$, and $B C=25$ and $A C=24$, what is $F C$ ?
23. In what base b is $12+34$ equal to 50 ?
24. At graduation, 21 people line up to receive their diplomas. The principal will not give a diploma to people in the kth place in line, where k is any multiple of 3 . What is the probability Silly Stanley will receive a diploma if he is at a random place in the line?
25. In triangle $A B C$ with $A B=6, B C=8$, and $A C=10$ an altitude is drawn from vertex $B$ to side $A C$ so that they meet at point $D$. What is the length of $C D$ ?
26. Every day there is a $40 \%$ chance of rain. What is the probability that at least one of Saturday or Sunday is sunny?
27. If the mean, median, and mode of $\{1,2,2, x, 4,4,11\}$ are all $x$, what is $x$ ?
28. What is the sum of the first twenty terms in the sequence $1,4,7,10 \ldots$ ?
29. Compute7!+6!7!8!6!.
30. A population of bacteria doubles in size every hour. If it begins with 7 microbes, after how many hours will the number of microbes be greater than 1000?
31. There are two trees that are each one foot tall. At the end of each day, the first tree doubles in height, while the second tree grows by ten feet. At the end of which number day will the first tree be taller than the second?
32. At the Gauss Math Supermarket, equations are available in packs of 4 or 5 . What is the largest number of equations one cannot purchase?
33. Find the sum of all integers $n$ such that $2 n^{\wedge} 2+21 n+32<n$.
34. Let r 1 and r 2 be the roots of the polynomial $\mathrm{x} 2+5 \mathrm{x}-17$. What is $\mathrm{r} 1^{\wedge} 2+\mathrm{r} 2^{\wedge} 2$ ?
35. A circle is inscribed within a square of side length 4 such that is tangent to all four sides. An equilateral triangle is then inscribed within the circle. What is the area within the square but outside the triangle?
36. How much wood could 9000 woodchucks chuck in 10 minutes if 3 woodchucks could chuck 7 wood every 21 minutes?
37. What is the remainder when 99950 is divided by 1000 ?
38. Andrew unrolls a hollow cylinder with a height of 24 cm and radius of $5 / \pi \mathrm{cm}$ into a rectangle. What is the area of the resulting (2D) shape?
39. How many 3-digit integers are divisible by either 19 or 20 ?
40. In the coordinate plane, Carl starts at ( 0,0 ). How many paths are there to the Gauss Math Supermarket at ( 4,4 ) if Carl can only move one unit right or up?

End of Sprint Round

## Gauss School and Gauss Math Circle

## 2019 Gauss Math Tournament

Grade 5-6 (Target Round 20 minutes, 8 Questions)

1. Lesser mage Antonida needs your help! She has forgotten the codes to her vault, but she remembers that her teacher, Benedictos, has left 2 clues:
2. The code is a 4 digit palindrome.
3. The sum of all the digits add up to 34 .

What is the sum of all possible codes?
2. Antonida completely coats a purple cube with orange paint. She then cuts up the original cube into 27 identical (smaller) cubes, picks a random piece and rolls it (like a die). What is the probability that the piece has an orange side facing up?
3. Antonida has an infinite sized flock of white and black sheep. At the beginning, $10 \%$ of the sheep are black. She can either cast a spell that doubles the number of black sheep (without changing the number of white sheep), or reduces the number of white sheep by the number of black sheep (it becomes zero if there are more black sheep). What is the minimum number of spells she must cast to have her flock be at least $90 \%$ black sheep?
4. Antonida's summoning hexagon is a regular octagon with side length 10. She must fit a summoning circle within the hexagon. What is the largest area summoning circle she can place within the hexagon?
5. The number 13A005697, where A is a digit, is the cube of a multiple of 3 . What is A ?

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6. Antonida has 14 magic fence posts. What is the largest area rectangular region she can inclose if there must be a fence post at every corner and there can be at most 2 feet between adjacent posts?
7. Antonida can no longer recall the ages of herself and her cat. However, she remembers that four years ago, she was four times as old as her cat, and twelve years ago, she was six times as old as her cat. How old is Antonida?
8. Antonida's Super Secret Series (SSS) consists of six positive integers in increasing order. Starting with 0 and ending with 25 , the first four terms form an arithmetic sequence, while the last three terms form a geometric sequence. What is the fifth term?

## End of Target Round

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Answer Key
Sprint

| 28 | 13 | 90 | 230 | 1 | 720 | 7 | 4 | 540 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 5 | $1 / 72$ | 2 sqrt(5 | 45 | 40 | 6 | 50 | 3600 | 4 | 10 |
| 45 | 31 | 6 | $2 / 3$ | $32 / 5$ | $21 / 25$ | 4 | 590 | $1 / 49$ | 8 |
| 6 | 11 | -25 | 59 | $16-3$ sqrt(3) | 10000 | 1 | 240 | 90 | 70 |

Target

| 18887 | $1 / 3$ | 5 | $(75+50$ sqrt(2)) $\pi$ |
| :--- | :--- | :--- | :--- |
| 5 | 48 | 84 | 15 |

