Set 1

1. **[3]** Larry's dad and Larry's dad's dad (i.e. Larry's grandfather) leave home for the local 7-Eleven, which is 12,000 kilometers from their house. Larry's dad drives for half the distance at 20 kilometers per hour, and half the distance at 30 kilometers per hour. Larry's dad's dad drives for half the time at 20 kilometers per hour, and half the time at 30 kilometers per hour. If they both leave home at the same time, how many hours later does Larry's dad dad arrive than Larry's dad's dad?

2. **[3]** Let $FATHER$ be a regular hexagon with side length 2. Let L be the center of the hexagon. Find the area of $LEFT$.

3. **[3]** How many ways are there to rearrange the letters in the string "DADLEFT" if the two Ds must be next to each other?

1.____

2.____

3.____

Set 2

4. **[4]** Curly was asked to compute the sum of the first N positive integers for some positive integer N. However, he accidentally omitted one of the first N positive integers, and arrived at the answer of 277. Which number did he omit?

5. **[4]** If a, b, and c are positive integers such that $a! \cdot b^c$ is divisible by 2019, what is the second-smallest possible value of $a + b + c$?

6. **[4]** The lengths of the altitudes of a triangle are 5, 6, and 10. Find the perimeter of the triangle.

4.____

5.____

6.____

Set 3

7. **[6]** Find the number of six-letter strings that have the following properties:

- Each letter is either A, U, G, or C, and
- The string "AUG" does not appear anywhere, i.e. there are no three consecutive letters that spell "AUG"

8. **[6]** Let $ABCD$ be a rectangle with $AB = 1$, and let P and Q be the feet of the perpendiculars from B and D to $\overline{AC}$, respectively. Suppose that the circle with diameter $\overline{PQ}$ is tangent to $\overline{AD}$ and $\overline{BC}$. Find BC.

9. **[6]** Joe has a unfair coin. If he flips the coin three times, the probability that he flips heads at most one time is six times the probability that he flips heads exactly two times. What is the probability that he flips heads exactly three times?

7.____

8.____

9.____

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Set 4

10. **[7]** Moe has four boxes labeled Box 1, Box 2, Box 3, and Box 4. How many ways are there for him to put five indistinguishable balls into the four boxes such that for each $1 \le i \le 4$, Box i contains at most i balls?

11. **[7]** Let a, b, and c be the three roots of the cubic

$$x^3 - x^2 + 2x + 7.$$

Compute

$$\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1}.$$

12. **[7]** Let (a_1, a_2, a_3, a_4) be a randomly chosen permutation of $(2, 3, 5, 8)$. Find the expected value of

$$(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_4)^2 + (a_4 - a_1)^2.$$

10.____

11.____

12.____

Set 5

13. **[8]** Find all pairs of real numbers (x, y) satisfying

$$x^2 - y^2 - 4x + 4y = 2019,$$
$$x^2 + 3y^2 - 4xy = -2021.$$

14. **[8]** Given that

$$10^{12} - 12^{10} = 93A,08B,635,776$$

for two (not necessarily distinct) digits A and B, find $10 \cdot A + B$.

15. **[8]** How many permutations $(a_1, a_2, ..., a_6)$ of $(1, 2, ..., 6)$ have the property that for each $2 \leq i \leq 5$, at least one of a_{i-1} and a_{i+1} is greater than or equal to a_i?

13.____

14.____

15.____

Set 6

16. **[10]** It is known that

$$\sum_{n=0}^{\infty} \frac{1}{n!} = e.$$

Evaluate

$$\sum_{n=0}^{\infty} \frac{n^2 - 2n + 3}{n!}$$

in terms of e.

17. **[10]** Let N be an integer, and suppose that there are exactly two integers K between 2 and 10 inclusive such that N cannot be expressed as the sum of K consecutive integers. Find the product of these two values of K.

18. **[10]** In ΔABC, $AB = 5$, $AC = 12$, and $BC = 13$. Points D and E lie on sides $\overline{AB}$ and $\overline{AC}$, respectively, such that $\overline{DE}$ is tangent to the incircle of ΔABC. If the area of ΔADE is $\frac{2}{3}$, find DE.

16.____

17.____

18.____

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Set 7

19. **[12]** Determine the number of ways to place four kings on a 4 by 4 chessboard such that no two kings attack each other.
(Two kings attack each other if the squares they occupy share a vertex).

20. **[12]** Let $ABCDE$ be a regular pentagon. A circle ω passes through the midpoints of $\overline{BC}$, $\overline{AD}$, and $\overline{AE}$. If the radius of ω is 5, find AC.

21. **[12]** Let b, y, e, p, and a be positive real numbers satisfying $b^5 + y^5 + e^5 + p^5 + a^5 = 793$. Find the smallest possible value of

$$(bye + 5)(pa + 32)$$

19.____

20.____

21.____

Answers

1. 20
2. $2\sqrt{3}$
3. 720
4. 23
5. 676
6 $\frac{15\sqrt{14}}{2}$
7. 3841
8. $\sqrt{3}$
9. $\frac{1}{64}$
10. 22
11. $-\frac{7}{3}$
12. 56
13. $(1012, 1011)$
14. 82
15. 32
16. $3e$
17. 32
18. $\frac{5}{3}$
19. 79
20. 10
21. 1152