Round 1

- 1.1 A tank of water is % filled. After 12 gallons have been drained, ½ of the water remains. How many gallons of water were in the tank originally?
- 1.2 In a math club, 40% of the students are girls and 60% of the students are boys. After 5 girls leave, the club is now 25% girls and 75% boys. How many students were in the math club before the girls left?

1.3 In triangle ABC, M is the midpoint of BC and N is the midpoint of AM. If the area of ABC is 56, what is the area of BCN?

Round 2

2.1 In triangle ABC, D is the foot of the altitude from A to BC. If AB=AC=115 and AD=92, what is CD?

2.2 What is the y-intercept of the parabola $\frac{x^2 + 4x + 3}{y - 1} = 3$?

2.3 Determine the sum of all values of x satisfying |x + |x + |x|| = 6.

Round 3

3.1 Compute $(1^2 - 29)(2^2 - 28)...(29^2 - 1)$.

3.2 A knight starts at (0,0). Every move, if it is at (x,y), it can move to either (x+2,y+1) or (x+1,y+2). How many points can be reached in exactly 3 moves?

3.3 What is the smallest possible value of 2^{x^2+2x+2} ?

Round 4

4.1 A regular hexagon has side length 1. Compute the product of the lengths of all the diagonals of this hexagon.

4.2 The sum of the digits of the base-7 representation of an integer is equal to the sum of the digits of the base-11 representation of the same integer. Find the smallest integer greater than 7 satisfying these properties. Express your answer in base 10.

4.3 How many numbers between 1 and 2017 inclusive are integer multiples of 4 or 5 but not 10?

Round 5

5.1 Let x and y be real numbers strictly between -1 and 1 such that $1 + x + x^2 + x^3 + \dots = y$, $1 + y + y^2 + y^3 + \dots = -6x$. Find x + y.

5.2 Rectangle ABCD has AB=6 and BC=12. Equilateral triangles ABE, BCF, CDG, and DAH are constructed such that E,F,G,H are outside ABCD. Determine the area of quadrilateral EFGH.

5.3 If x and y are real numbers satisfying x + y = 1 and $x^3 + y^3 = 4$, determine $x^2 + y^2$.

Round 6

6.1 An ant starts at (0,0). The ant randomly moves 1 unit up, down, left, or right 2 times. Let P(x,y) denote the probability that the ant is on (x,y) after the two steps. Determine the average value of P(x,y) over all (x,y) such that P(x,y)>0.

6.2 If x,y,z are real numbers such that $(x + y)^2 = 1$, $(y + z)^2 = 4$, and $(z + x)^2 = 9$, how many possible values are there for x?

6.3 Isosceles trapezoid ABCD has AC=BD=5 and area 12. What are all of the possible values of its height?

Round 7

7.1 For how many integers k are there no real solutions to the system of equations $x^2 - xy = k^2 - 3k_y^2 - xy = k - 99?$

7.2 Let (a,b,c,d,e,f) be a permutation of (1,2,3,4,5,6). What is the expected value of |b-a| + |d-c| + |f-e|?

7.3 Let $f(x) = x^2 + 2x + 2$. There are two real solutions to the equation f(f(f(x))) = 0. Determine their positive difference.

Solutions
1.1: 30
1.2: 25
1.3: 14
2.1: 69
2.2: 2
2.3: -4
3.1:0
3.2: 4
3.3: 2
4.1: 216
4.2: 35
4.3: 601
5.1: 1/6
5.2: $24 + 15\sqrt{3}$
5.3:3
6.1: 1/9
6.2: 5
6.3: 3 and 4
7.1:21
7.2: 7
7.3: 2∛2