Gauss School and Gauss Math Circle

2020 Gauss Math Tournament

Division II (Sprint Round 50 minutes, 40 Questions)

1. What is $\frac{62}{2} + 29$?
2. What is the perimeter of a regular hexagon with side length 5?
3. What is the mode of the following set of numbers: {1,3,4,1,5,1,7,1,3,1,2,3,1}
4. What is the next term in the sequence: 2, 3, 5, 8, 12, _?
5. A three-digit number beginning with a 3 also contains the digits 8 and 5. What is the probability that it's divisible by 7?
6. Suzy has 3 cows. If each of her cows can be sold for 25 cents, how many dogs can she buy if each dog costs 7 cents (you cannot buy fractional amounts of dogs)?
7. What is the absolute value of the difference between the sum of the interior angles and exterior angles of a triangle?
8. A rectangle's length is 3 times its width. If the area of the rectangle is 48, what is the perimeter?
9. Spencer orders two scoops of ice cream. The available flavors are mint chocolate chip, cotton candy swirl, pistachio, and caramel toffee. How many distinct combinations are possible (note: one scoop is on top of the other)?
10. A compulsive liar lies with ½ probability. A compulsive liar tells three statements, what is the probability that 2 of them are lies?
11. What is the sum of the divisors of 323 (include 323 and 1 as divisors)?
12. What is 2+4+6+...+50?
13. The operation <a> is equal to $\frac{a^3}{(a+2)^3}$. What is <1>*<3>*<5>*<7>*<9>?
14. What is the slope of the line through (1, 2) that is parallel to 3x+4y=7?
15. What is 5 percent of 75 percent of 800?
16. What is the unit digit of 6^{2020}?
17. An isosceles right triangle has hypotenuse 12. What is the area?
18. How many prime numbers between 1 and 1000 are divisible by both 17 and 3?
19. How many digits are in $10^{28} \times 100^4$?

20. What is the unit's digit of 23^{24}?
21. How many numbers n are 5n and $\frac{n}{5}$ both positive 4 digit integers?
22. A rectangular prism has volume 40. If the length is increased by 20%, the width is decreased by 20% and the height is increased by 25%, what is the volume of the new prism?
23. What is the sum of all the roots of $x^3 - 3x^2 + 2x$?
24. What is the absolute value of the difference between the quotient and remainder of 113 divided by 13?
25. What is the largest three-digit multiple of 3 that has a units digit of 7?
26. Tom needs to mop his house. He can do this in two hours by himself. Today, Tom invites his good friend Huck to mop the house with him to make it less boring. Huck can mop a house in three hours by himself. How long does it take for both boys to mop the house together?
27. What is the 2nd largest possible sum of two three-digit numbers that use the digits 1,2,4,5,7,8 (no digit may be repeated)?
28. Brian likes solving math problems. Each day, Brian solves an integer number of problems in an integer amount of hours. On a sunny day where the ice cream truck does not come, Brian solves 10 problems an hour. On days when the ice cream truck comes, Brian solves 15 problems an hour so he can go out to eat some ice cream. On rainy days where the ice cream truck does not come, Brian solves 6 problems an hour because he is sad. What is the least possible value of n?
29. What is the largest prime factor of C(250, 125)? (Here C(250, 125) means 250 choose 125: $\frac{250!}{125! \times 125!}$.
30. Mr. B, a math teacher, is tasking his students in drawing a rectangle with integer side lengths. He wants the perimeter of the rectangles to be 80 units. What's the difference between the area of the largest rectangle and the smallest rectangle that the students could draw?
31. How many perfect squares evenly divide 12!?
32. Triangle ABC is a non-degenerate triangle with all integer side lengths. We have AB = 8 and AC = 10. A person chooses an integer value for BC. What is the

probability that triangle ABC is an obtuse triangle? (Here, non-degenerate means that the triangle is not a line.)

33. Currently, Oswald is at the point (2, 2) and wishes to go to the library at point (4, 5). Oswald can either move right or upwards one unit. Unfortunately, Oswald is a bit daft and in his path to the library, he goes down one unit exactly once to follow a butterfly. How many possible ways are there for Oswald to get to the library in 7 moves (it is okay if Oswald gets there earlier but his location must be (4,5) at move 7)?
34. Let circles A and B be externally tangent and internally tangent to C. Circles A and B both have radii half of the radius of C. What is the ratio of the area inside circle C but outside circles A and B to the area of circle C?
35. Let triangle ABC have side lengths AB = 3x, BC = 5x, and CA = 7x. The area of triangle ABC is $15\sqrt{3}$. What is the semiperimeter of ABC?
36. Find the sum of the 2 and 3-digit numbers that have their sum of digits equal to 2.
37. One day, Kevin the frog is sitting on a lilypad in the middle of a pond, and he wants to reach the land. Every day, he paddles towards the nearest land and is able to make it 4 meters. Every night, miniature currents take him back 3 meters towards the center of the circular pond. If Kevin is able to jump to land once he is within 5 meters, and he starts in the middle of a circular pond with a radius of 25 meters, how many nights has Kevin endured?
38. How many ways can you stack 2 math books, 2 science books, 2 history books, and 2 English books such that all the math books are above all the science books, and all the history books are above all the English books? All books are distinguishable.
39. McDonalds offers 2 options for chicken nuggets: 4-piece set and a 7-piece set. What is the largest number of chicken nuggets that I cannot get by buying combinations of 4 and 7-piece sets?
40. How many paths of length 5 are there from (0,0) to (3,2) that don't pass through the point (1,1)?