

Gauss School and Gauss Math Circle
2020 Gauss Math Tournament
Division III (Target Round 20 minutes, 8 Questions)

Target Round

1. Let $f(x, y) = x^3 + 3xy^2$. What is $f(20, 12) - f(12, 20)$?
2. Chika is cheating at a 20 question, true/false trivia contest with Miyuki. She always answers every question correctly, but Miyuki has a 50% chance of answering a given question correctly. Miyuki starts with 0 points, but Chika starts with -5 points as punishment for being a dirty cheater. The probability that Chika ends with fewer points than Miyuki is $\frac{m}{2^n}$ for positive integers m, n where m is odd. Find m .
3. Find the 100th smallest positive integer relatively prime to 2020.
4. We call a positive integer *clean* if the sum of its divisors is odd. How many divisors of 2016 are clean?

5. Compute

$$\sum_{k=2}^{20} \frac{1}{k^2 - 1}.$$

6. How many subsets S of $\{1, 2, 3, 4, 5, 6, 7\}$ have the property that the sum of the elements of S is 14?
7. There are five major cities in the Kingdom of Lugnica (four of which are Lugnica, Priestella, Picou-tatte, and Flanders), with a road connecting each pair of these cities. The Witch of Envy wishes to destroy some of these roads so that afterwards, there is a city which is adjacent to an odd number of roads. How many ways can she do this?
8. Snorlax has three stones in the plane and may perform the following operation. First, he chooses two different stones A and B . He may then move the stone B to a new location C such that C lies on AB . Originally, his three stones form an isosceles right triangle. Let M be the fewest number of operations Snorlax needs to make so that the three stones form an equilateral triangle. Find the number of sequences of M moves that take Snorlax's stones to the vertices of an equilateral triangle.