

Gauss School and Gauss Math Circle

2018 Gauss Math Tournament

Grade 7-8 (Sprint Round 50 minutes, 40 Questions)

1. How many ways are there to arrange the letters in “MEME?”
2. The capacity of a school is 1100 students and the current enrollment is 980 students. If the student population increases at a rate of 5 percent per year, in how many years will the enrollment exceed the capacity?
3. The length of each edge of a cube is decreased by 40%. By what percent does the volume of the cube decrease? Express your answer to the nearest tenth.
4. Assuming that the birth of a boy or a girl is equally likely, what is the probability that the three children in a family include at least one boy and one girl? Express your answer as a common fraction.
5. The area of rectangle ABCD with vertices A(0, 0), B(0, 4), C(x, 4) and D(x, 0) is 28 square units. If $x > 0$, what is the value of x?
6. Two different numbers are randomly selected from the set $\{1, 2, 3, 4\}$ and they are multiplied. What is the probability that the product is even? Express your answer as a common fraction.
7. A diagonal of a polygon is a segment joining two non-consecutive vertices of the polygon. How many diagonals does a regular octagon have?
8. How many different sets of three numbers, each number chosen from the integers 1-16, could be the side lengths of a right triangle?
9. To take quizzes, each of 30 students in a class is paired with another student. If the pairing is done randomly, what is the probability that Margo is paired with her best friend, Irma? Express your answer as a common fraction.
10. A regular octagon and decagon have the same perimeter. Find the ratio of the octagon's side length to the decagon's.
11. Define a & $b = a^2 - a + 2ab$. Evaluate $1000 \& ((5 \& (-2)))$.
12. What is the smallest positive square I can multiply by 300 to make it a perfect cube?
13. Samuel can run at 12 mph, and Josh can run at 15 mph. How much faster can Josh run 120 miles, in minutes?
14. On the planet Gauss, Ernests have 3 heads, 2 tails and 3 legs, Cows have 1 head, 1 tail, and 4 legs, and Snakes have 1 head, 1 tail, and 0 legs. If there are a total of 23 heads, 18 tails, and 39 legs, how many total animals are there?
15. A snail is crawling up from the bottom of a 100 meter well. On day N, it will crawl 2N meters, and on night N, it will fall N meters while sleeping. On which day will it reach the top of the well?
16. Compute the sum of the positive prime factors of 1000.
17. What is the remainder when the sum $1^2 + 2^2 + 3^2 + \dots + 10^2$ is divided by 11?

18. An ant is traveling on the surface of a cube with side length 1. What is the smallest distance that the ant has to travel to go from one vertex to the opposite vertex?
19. What is the probability that a randomly-chosen three-digit number contains the digit 4 exactly twice? Express your answer as a common fraction.
20. Lola drives from her house to the park at a rate of 30 mph, and drives home at a rate of 40 mph. What is her average speed throughout this trip?
21. Determine the area in square units of parallelogram ABCD with vertices A (-6, 2), B (12, 2), C (15, 8) and D (-3, 8).
22. A rectangle has a perimeter of 30 units and its dimensions are whole numbers. What is the maximum possible area of the rectangle in square units?
23. LeJames Bron has a 75% chance of making any single shot. Given that he has already made 5 shots in a row, what is the probability he'll make a 6th in a row? Express your answer as a percentage.
24. 8 friends named A,B,C,D,E,F,G and H are lining up in a queue. If A must be in front of B, B must be in front of C, and D must be in front of E, how many ways are there to line them up?
25. A haunted house has 30 2-way doors on the outside. How many ways can William go in and come out through different doors?
26. William attempted to sum the first n numbers, however he missed a number b, and got 2018 instead. Find the ordered pair (n,b).
27. A pair (x,y) is called good if $(x-y)^3 = x^3 - y^3$ and $0 \leq x, y \leq 10$. Find the sum of all possible products xy.
28. How many noncongruent triangles are there which have altitudes with lengths 5, 6, and 7.
29. A palindrome is a number which is the same read forwards or backwards, like 12321. How many more 8-digit palindromes are there compared to 7-digit palindromes?
30. In triangle ABC, AB=8, AC=15, and BC=17. Compute the length of the altitude from A.
31. Ernest is giving candies to his students so they will be happy. When he gives N to each student, one student is without candy. When he gives N-1 candies to each student, there is enough for three more students. If he has more than 10 students, what is the least amount of students he can have?
32. Evaluate the sum: $\sum_1^{999} 3n^2 + 3n + 1$
33. Triangle $A_0B_0C_0$ has $AB_0 = 12, AC_0 = 13, B_0C_0=5$. If I define B_iC_i to be the line parallel to $B_{i-1}C_{i-1}$ and tangent to the incircle of $AB_{i-1}C_{i-1}$, find the length of B_4C_4 .
34. There are 100 light switches in a row, numbered 1 to 100. Person 1 will come over and turn on any light which is NOT divisible by 1, then person 2 will come and change the state of any light which IS divisible by 2. This continues, with the odd numbered people changing the state of any light which is not divisible by their number, and opposite for evens. After Person 100 has left, how many lights are now on?

35. In Yee-land, a word consists of a string of Ys and Es. A word is called Yee-tastic if it contains a block of 3 letters which spells out "YEE." How many Yee-tastic words have 10 letters?
36. What is the minimum number of US coins I need to be able to make any coin value from \$0.01 to \$0.99 with some combination of the coins?
37. Triangle ABC has $AB=13$, $AC=15$, and $BC=14$. If $D \neq A$ is on segment AC such that $BD=13$, find the ratio of the inradius of ABD to the inradius of BDC.
38. Helen wants to make a Wheel of Fortune with 9 sectors of equal size. She wants three of them to be red, three to be green, and three to be blue. Given that configurations are distinct up to rotation, how many ways can she do it?
39. Ernest writes the numbers 1 through 9 on the board. Then, he takes two numbers a and b , erases them, and writes $ab+a+b$ on the board in their place. He repeats this until there is only one number left on the board. Find the maximum possible last value.
40. Find the rightmost three digits in the decimal expansion of 2018^{2018} .

End of Sprint Round

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Grade 7-8 (Target Round 20 minutes)

1. A weighted coin is flipped 3 times, and has equal probability of displaying all heads as it does displaying exactly one tails. What is the probability on a single throw that it will be heads?
2. Consider the polynomial $P(x)=x^3-2018x-23$, with roots r_1, r_2 and r_3 . Compute $r_1^3+r_2^3+r_3^3$.
3. Trapezoid ABCD has BC perpendicular to AB and CD, and $AB+CD=BC=3$. There exists a semicircle with diameter on AD which is tangent to BC and CD. If $0.5<AB<CD$, find the maximum radius of such a semicircle
4. There is a rational number x such that $(\frac{9}{4})^x = x^{9/4}$, and $x \neq \frac{9}{4}$. What is the value of x ?

5. Compute the infinite sum show below:

$$\sum_{n=1}^{\infty} \frac{n}{n^4 + 4n^2 + 16}$$

6. It is known that the number 1000243729 is the product of two primes. What is the smaller of the two prime factors?
7. William wants to get from (0,0) to (5,5) in the coordinate plane. However, his only possible moves are to go up one unit, right one unit, or diagonally down-right one unit in each direction. How many move sequences will get him to his destination if he cannot cross the axes?
8. Let x be a positive real number such that $[x]^4 - 3x^3 - [x - 1]^2 + 2 = 0$. Let P be the product of all possible values of x . Find $[P]$.

End of Target Round

Sprint Round Answers

1. 6
2. 3
3. 78.4%
4. $\frac{3}{4}$
5. 7
6. $\frac{5}{6}$
7. 20
8. 4
9. $\frac{1}{29}$
10. $\frac{5}{4}$
11. 999000
12. 90000
13. 120
14. 13
15. 13
16. 7
17. 0
18. $\sqrt{5}$
19. $\frac{13}{450}$
20. $\frac{240}{7}$
21. 108
22. 56
23. 75%
24. 3360
25. 870
26. (64,62)
27. 385
28. 1
29. 0
30. $\frac{120}{17}$

31. 13
32. 999,999,999
33. $\frac{80}{81}$
34. 10
35. 792
36. 10
37. $\frac{264}{49}$
38. 188
39. 3628799
40. 424

Target Round Answers

1. $\frac{3}{4}$
2. 69
3. $\frac{3}{2}$
4. $\frac{27}{8}$
5. $\frac{7}{48}$
6. 709
7. 15052
8. 17