2022 Gauss Math Tournament Target Round (Div. 3)

June 11, 2022

1. What is the sum of the digits of 1111^2?

2. Seven trees are equally spaced along one side of a straight road. The distance from the first tree to the fifth is 60 feet. What is the distance in feet between the first and last trees?

3. A typical can of soda holds 12 ounces of soda. What is the minimum number of whole cans needed to fill a 1 liter (33.814 oz) jug?

4. A goal in Gauss ball can either count for 216 points or 337 points, what is the remainder when greatest point value where it is not possible for a team in Gauss ball to end with is divided by 1000?

5. Evaluate $\sqrt{11 - 6\sqrt{2}} + \sqrt{11 + 6\sqrt{2}}$.

6. In $\triangle ABC$, $AB = 8$, $BC = 6$, $CA = 10$, and D is the midpoint of the B-altitude. Ray CD meets the circumcircle of $\triangle ABC$ at another point E. If the length of DE can be expressed as $\dfrac{a\sqrt{b}}{c}$ in simplest radical form, find $a + b + c$.

7. Two circles α and β, each with radius 5, have centers A and B, respectively, and they intersect at M and N. Extend segment AB to intersect with α at X and β at Y (X and Y are not on line segment AB). Given that $XY = 12$, the length of MN can be expressed in simplest form as $a\sqrt{b}$. What is $a + b$?

8. While watching a show, Ayako, Billy, Carlos, Dahlia, Ehuang, and Frank sat in that order in a row of six chairs. During a break, Frank left his seat to get a snack, leaving one of the seats empty. Each second during the break, one randomly chosen person sitting adjacent to the empty seat will get up and sit in it. Compute the expected amount of time, in seconds, for Ayako's seat to become empty for the first time.