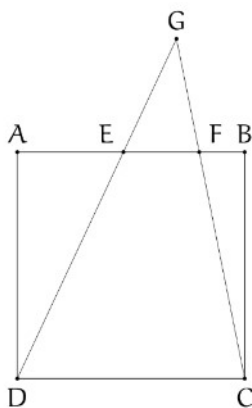


# 2023 Gauss Math Tournament Sprint Round (Div. 2)

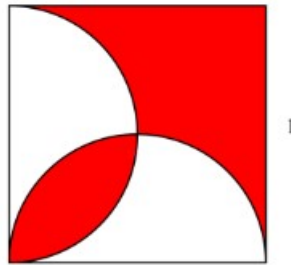
June 10, 2023

1. Caroline earns a multiple of \$20,000 from scholarship through an art contest, yet she also makes a perfect square amount of dollars. She can just do one of these moneymaking activities or do both, and how many ways can she make \$1,000,000?
2. How many solutions, in which  $a$ ,  $b$ , and  $c$  are integers, are there to  $|a - 6| + 2|b - 3| + 3|c - 2| = 10$ ?
3. A number generator multiplies the input by 2 when Ava pushes the up button and raises the input to the 3rd power when she pushes the down button. If Ava starts at 1 and randomly chooses which button to press, what is the probability that Ava will reach 8?
4. In a bowl consisting of 10 distinct coconuts, with 7 good ones, Elie picks out 3, of which at least 1 is good. How many ways can she do this?
5. Square ABCD has side length 12. E and F are on AB such that  $EF=4$ . What is the area of triangle CGD?



6. Convert  $1101_2$  into decimal (base 10).
7. It takes Alan 4 hours to mow 10 acres and his son 6 hours to mow 12 acres. How many hours will it take both of them to mow a lawn of 36 acres?
8. Calculate  $999 + 999 \times 999$ .
9. If 5 basketballs and 3 volleyballs cost 100 dollars and 2 basketballs and 3 volleyballs cost 67 dollars, how much does the basketball cost?
10. Port A and Port B are 252 miles away from one another. A boat starting at Port A goes along the current and arrives at Port B in 9 hours. On its return trip, it goes against the current, and takes 14 hours. What is the speed of the boat in miles per hour?
11. Ali Baba has 7 coins and he knows that one of them is a counterfeit, meaning it is lighter than the others. If he only has a balancing scale, at least how many weighings does Ali Baba need to determine the counterfeit coin?

12. Vladimir, Leon, and Joseph were practicing baseball in the school courtyard, and one of them accidentally broke a window when throwing the baseball too hard. When the principal asked them who did it, they replied in the following ways: Vladimir said, "Leon did it." Leon said, "I did not do it." Joseph said "I also did not do it."  
If only one person told the truth, who broke the window?
13. The six digit number  $a2004b$  is divisible by 99. What is the six digit number?
14. The sum of the reciprocals of 3 prime numbers is  $\frac{311}{1001}$ . What is the sum of those 3 prime numbers?
15. An old man said, "If you take my age and add 14, divide by 3, subtract 26, and then multiply by 25, you get exactly 100." How old is the old man?
16. Is  $1+2+3+\dots+100+101$  odd or even?
17. How many ways are there to put 7 identical balls into 4 baskets such that each basket has at least 1 ball?
18. If the side length of the square is 1, what is the area of the shaded region?



19. What is  $\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \dots \times \frac{99}{98} \times \frac{100}{99}$ ?
20. Let  $x$  and  $y$  be integers greater than 1 such that  $xy = 184$  and  $x$  is a factor of  $y$ . What is  $y/x$ ?
21. What is the equation of the line that perpendicularly bisects (splits the line in half at a  $90^\circ$  angle) line segment  $AB$  if  $A$  is  $(3, 0)$  and  $B$  is  $(-1, 2)$ ? Answer in y-intercept form ( $y = mx + b$ ).
22. If we flip the digits of a two digit number, we get 36 more than the original problem. The sum of the digits of the number is 8. What is the flipped number?
23. If an equilateral triangle with length 6 is inscribed in a circle, what is the circumference of the circle divided by  $\pi$ ?
24.  $2^{4x-1} = 8^{x+6}$ . What is  $x$ ?

25. January 1st, 2023 was a Sunday. What will be the next month that has its first day on a Sunday?
26. Add the numbers  $425_7$ ,  $32_5$ , and  $57_8$ , then express the sum in base 2.
27. From the numbers 0, 1, 2, 3, 4, 5, 6, 7, select 6 numbers to form a six-digit number without repeating digits. This six-digit number must be divisible by 63. What is the maximum value of this six-digit number?
28. Suppose you have 12 non distinguishable apples to be placed into 3 identical boxes. Each box must contain at least 3 apples. How many different ways are there to arrange the apples?
29. There are 5 red marbles, 3 blue marbles, and 2 green marbles in a bag. If two marbles are randomly drawn from the bag without replacement, what is the probability of selecting a red marble followed by a blue marble?
30. Sam walks 1.2 meters south from point A, then walks 1 meter east, followed by another 1.8 meters south, then walks 2 meters east, and finally walks 1 meter south to reach point B. What is the distance in meters between point B and point A?
31. A cube with an edge length of 10 centimeters is cut into smaller cubes each with an edge length of 2.5 centimeters. How much greater is the surface area of the original cube than the total surface areas of the smaller cubes?
32. Mixing 80 grams of a 25% concentration sugar solution with 60 grams of a 15% concentration sugar solution, how many grams of sugar should be added to obtain a 44.5% concentration sugar solution?
33. Bob is the new manager of a movie theater. He decides to lower the price of each movie ticket by 3 dollars. It turns out that the audience number of every screening increases by 50% and the revenue increases by 20%. What was the original price of the tickets, before the change?
34. What is the sum of the interior angles of a nonagon?
35. What is the area of a regular hexagon with side length 2?
36. A circle's radius is 5. What is the area of a triangle whose longest side is the diameter of a circle and has all of its vertices on the circle?
37. One of the factors of 2023 is 7. What is the smallest positive number that, when multiplied by 2023, will yield a perfect cube?
38. What is the least possible value of  $a+b$  if  $58_a = 85_b$ ?
39. What is the probability that a subset of  $\{1, 2, 3, 4, 5, 6, 7, 8\}$  contains 2 or 3 but not both 2 and 3?

40. In a game of Among Us, there is one task that requires a player to match 4 differently colored wires to another 4 colored wires, with corresponding colors for matching. If a crewmate randomly matches the wires, what is the probability that the task is successful?