Name ________________________

1. _______________

2. _______________

3. _______________

1. [3] I flip a fair coin four times. What is the probability that I get more head than tails?

2. [3] The Fibonacci sequence is 1, 1, 2, 3, 5, 8, ... Each number after the first two numbers is the sum of the preceding two numbers. What is the first perfect square greater than 1 to occur in this sequence?

3. [3] Al, Barb, Cal, Di, and Ed were doing fund raising for charity. In total, they raised $150. But each of them raised unequal amount of money. They would have raised the same amount if Barb gave half of the money she raised to Al, then Cal gave 1/3 of the money he raised to Barb, then Di gave ¼ of the money she raised to Cal, then Ed gave 1/6 of the money he raised to Di. How many dollars did Al actually raise?

Name ___________________________

4. ________________

5. ________________

6. ________________

4. [4] Find the sum of the digits of the integers from 1 to 100 inclusive.

5. [4] Solve for x: $\sqrt{x-3} + \sqrt{x+7} = \sqrt{x+39}$

6. [4] If the numbers 63+k, 118+k, 175+k, and 234+k are four consecutive perfect squares, find k.

Name _________________________

7. _______________

8. _______________

9. _______________

7. [6] A bag contains red balls and white balls. If five balls are to by pulled from the bag, with replacement, the probability of getting exactly three red balls is 32 times the probability of getting exactly one red ball. What percent of the balls originally in the bag are red?

8. [6] What is the probability that, when two different integers are chosen at random from the first 100 positive integers, their sum is even? Express your answer as a fraction.

9. [6] Points X and Y lie on sides AB and AC respectively of triangle ABC. If AB=7, AC=10, AX=4 and AY=6, what is the ratio of the area of triangle AXY to the area of triangle ABC?

Name ___________________________

10. ________________

11. ________________

12. ________________

10. [7] Find the remainder when $10^{2018} + 10^{201} + 10^{20} + 10^{2}$ is divided by 7.

11. [7] Let AB be the diameter of a circle, where O is the center. Let C be a point on the circle such that AC is 6. If the radius of the circle is 5, find the distance from O to AC.

12. [7] Find the product of all solutions, not necessarily real, to: $\sqrt{1+x} + \sqrt{1-x}$ $= \sqrt{2+2x} * \sqrt{2-x}$ +x

Name ________________________

13. ______________

14. ______________

15. ______________

13. [8] In triangle ABC, AC=BC=5 and AB=6. Let ω be the circle tangent to AB and AC whose center lies on BC. Compute the radius of ω.

14. [8] Given that a, b, and c are positive reals with $a+b+c=3$ and $a+ab+2abc=9/2$, find $100a+10b+c$.

15. [8] A classroom has 7 girls and 4 boys. The teacher wants them to line up in height order, which they do successfully without problem. However, if two boys end up next to each other, they start playing Fortunate Duos together and the teacher will get mad. What is the probability that the teacher will not get mad?

Name ________________________

16. _______________

17. _______________

18. _______________

16. [10] Let ABC be an isosceles triangle with AB=AC. Let D, E, and F be the feet of the altitudes from A, B, and C respectively. Extend CF to meet the circumscribed circle of ABC at G. If the measure of angle B is 75 degrees, find the angle (in degrees) subtended by minor arc GAC.

17. [10] Triangle $A_0B_0C_0$ has $AB_0 = 12, AC_0 = 13$, B_0C_0=5. If I define B_iC_i to be the line parallel to $B_{i-1}C_{i-1}$ and tangent to the incircle of $AB_{i-1}C_{i-1}$, find the length of B_4C_4.

18. [10] A pair (x,y) is called good if (x-y)^3=x^3-y^3 and 0≤ x,y≤10. Find the sum of all possible products xy.

Name ________________________

19. _______________

20. _______________

21. _______________

19. [12] Let Γ be a circle with diameter BD, and ABC be an equilateral triangle such that C lies on BD. Suppose that Γ intersects AB at two distinct points B and P and AC at Q. Given that $AQ/QC = \frac{1}{2}$, find AP/PB.

20. [12] Let ABC be a 3-4-5 triangle with right angle at B. Let D, E, F be the midpoints of BC, CA, AB respectively. Find the area of a triangle with side lengths AD, BE, and CF.

21. [12] I have a graph on 10 vertices with n edges. If I remove one (specifically chosen) edge, I get a graph with no triangles in it. What is the maximum value of n?

Answer Key:

1. 5/16
2. 144
3. 11
4. 901
5. 11
6. 666
7. 80%
8. 49/99
9. 12/35
10. 5
11. 4
12. 0
13. 24/11
14. 321/2
15. 7/33
16. 90
17. 80/81
18. 385
19. 5/7
20. 9/2
21. 26