Name ________________________

1. ______________

2. ______________

3. ______________

1. [3] Compute the value of

$$\binom{6}{0}^2 + \binom{6}{1}^2 + \binom{6}{2}^2 + \binom{6}{3}^2 + \binom{6}{4}^2 + \binom{6}{5}^2 + \binom{6}{6}^2$$

2. [3] While adding up the first n positive integers, a student accidentally omitted the average of these integers from the sum. If the sum the student obtained is 420, what is the value of n?

3. [3] Jack rolls three standard dice. What is the expected value of the product of the numbers that he rolls?

Name ___________________________

4. ________________

5. ________________

6. ________________

4. [4] In ΔABC, $AC = BC = 5$ and $AB = 6$. Let ω be the circle tangent to $\overline{AB}$ and $\overline{AC}$ whose center lies on $\overline{BC}$. Compute the radius of ω.

5. [4] Find the number of four-digit multiples of seven containing exactly two distinct digits.

6. [4] Find the largest prime factor of $3^{12} + 8^2$.

Name ___________________________

7. ________________

8. ________________

9. ________________

7. [6] Let a, b, c, and d be positive integers satisfying $ab + bc + cd + da = 666$. Find the smallest possible value of $a + b + c + d$.

8. [6] Let m and n be positive integers which satisfy m3+6=2n.
Find the number of ordered pairs (m, n) such that m and n are both less than 2018.

9. [6] Find the remainder when $10^{2018} + 10^{201} + 10^{20} + 10^{2}$ is divided by 7.

Name ______________________

10. ______________

11. ______________

12. ______________

10. [7] If $\gcd(a,b)^2+ab=12*\gcd(a,b)$, let N be the product of all possible values of a, where a and b are positive integers. Find the last three digits of N.

11. [7] Let ABC be a 3-4-5 triangle with right angle at B. Let D, E, F be the midpoints of BC, CA, AB respectively. Find the area of a triangle with side lengths AD, BE, and CF.

12. [7] Let S be the set of points in the coordinate plane satisfying the following conditions:

1) $y \geq 0$.

2) $x^2 + (max(x,y))^2 \leq 2018$.

Find the area of S.

Name ________________________

13. _______________

14. _______________

15. _______________

13. [8] Let a,b,c be complex numbers such that a+b+c = 1, a^2 + b^2 + c^2 = 2, a^4+b^4+c^4 = 4. Find the value of a^6+b^6+c^6.

14. [8] Let f(x) be a polynomial such that the remainders when f(x) is divided by x-1, x+1, and x^2 + 1 are 17, 9, and 7, respectively. Let r(x) be the remainder when f(x) is divided by x^4-1. Calculate the value of r(4).

15. [8] Let Γ be a circle with diameter $\overline{BD}$ and ΔABC be an equilateral triangle such that C lies on $\overline{BD}$. Suppose that $\overline{AB}$ intersects Γ at two distinct points B and P, and $\overline{AC}$ intersects Γ at Q. If $\frac{AQ}{QC} = \frac{1}{2}$, find $\frac{AP}{PB}$.

Name ___________________________

16. ________________

17. ________________

18. ________________

16. [10] Triangle ABC has AB=5, AC=8, BC=7. If the foot of the A-altitude is D, and E,F are the reflections of D over AB and AC respectively, find the area of AEF.

17. [10] In triangle ABC, let D, E, and F be the feet of the altitudes from A, B, and C respectively. Let AD intersect EF at point X. If AE = 6, AC=16, and AF=32/5, find the ratio of FX to EX.

18. [10] Let r_1, r_2, and r_3 be the three distinct roots of the polynomial $x^3 + 500x - 1009$. Find the integer closest to the value of $|r_1| + |r_2| + |r_3|$.

Name ________________________

19. _______________

20. _______________

21. _______________

19. [12] In ΔABC, $\angle A = 90°$, and D and E are on $\overline{BC}$, with D between B and E, such that $\overline{AD}$ and $\overline{AE}$ trisect $\angle A$. If [ACE]=10 and [ADE]=20, find [ABC].

20. [12] Find the sum of all prime numbers p such that $p^2 - 3p + 8$ is the cube of an integer.

21. [12] [10] Let a and b be real numbers, and r_1, r_2, r_3, r_4 be the roots of the polynomial $x^4 + ax^3 + x^2 + bx + 1$. Suppose that
$(r_1 + 1)(r_2 + 1)(r_3 + 1)(r_4 + 1) = -18$,
$(r_1^2 + 1)(r_2^2 + 1)(r_3^2 + 1)(r_4^2 + 1) = 420$.
Find the product of all possible values of a.

Answer Key:

1	924
2	29
3	343/8
4	24/11
5	75
6	37
7	55
8	0
9	5
10	480
11	9/2
12	(2018+3027pi)/4
13	515/64
14	194
15	5/7
16	300sqrt(3)/49
17	344/375
18	47
19	70-20sqrt(3)
20	46
21	11/2